Adaptive Parameter Estimation for Microbial Growth Kinetics

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An adaptive parameter estimation algorithm for a class of biochemical processes expressed by a nonlinearly parametrized Monod's growth kinetics model is presented. Contrary to conventional least-square or gradient-type identification techniques, the proposed parameter estimation algorithm is developed based on Lyapunov's stability theory. A novel class of parameter-dependent Lyapunov functions is utilized to remove the difficulty associated with estimating the unknown parameters that appear nonlinearly. A persistence of excitation (PE) condition is investigated to guarantee the convergence of the estimation scheme. Simulations are provided to verify the effectiveness of the new approach and the theoretical discussion.

Introduction

Modeling and parameter identification are normally the prerequisites for the development of optimization and control methods. The research on parameter estimation of linear dynamic systems has been an active area over the last 30 years (Astrom and Eykhoff, 1971; Boyd and Sastry, 1983; Goodwin and Sin, 1984; Astrom and Wittenmark, 1989; Ioannou and Sun, 1996). It has been shown that if system signals are persistently exciting, that is, the signals contain sufficient frequency contents, then the estimated parameters are guaranteed to converge to their true values. Recently, the parameter convergence problem has been investigated for adaptive control design of a class of nonlinear systems (Zhang et al., 1996; Lin and Kanellakopoulos, 1998). A procedure has been developed to determine whether a reference signal is sufficiently rich for a given nonlinear plant *a priori*. An interesting result obtained in the work (Lin and Kanellakopoulos, 1998) is that the presence of the nonlinearities in the plants weakens the PE condition, and, thus enhances the parameter convergence of nonlinear adaptive control systems. In the references (Krstic, 1996; Li and Krstic, 1998; Krstic and Deng, 1998) invariant manifold theory is used to investigate parameter convergence issues in the presence of persistent excitation, partial excitation, and the least excitation for a class of parametric strict-feedback nonlinear adaptive systems. The results show that, in the absence of PE, an adaptive stabilization mechanism can result in the convergence of parameter estimates to destabilizing values. In control systems, parameter convergence is important for overall system stability and robustness. The search for PE conditions for different nonlinear systems has been an important and challenging topic in the fields of parameter estimation and control.

During the last three decades, model identification and parameter estimation of chemical processes have been of great interest to researchers and practitioners (Ray and Aris, 1965; Spriet, 1982; Holmberg and Ranta, 1982; Holmberg, 1983; Dochain and Bastin, 1984; Bastin and Dochain, 1986). In the design of estimation algorithms, an appropriate parameterization of plants is important, since some models may be more convenient than others. The most commonly encountered models in the literature are linear (or affine) in the unknown parameters. The main reason for this is that linearly parameterized plants lead to simpler design and analysis of parameter estimation algorithms. Nevertheless, many modeling and estimation problems encountered in practice involve nonlinearly parameterized models. This is the situation in a number of biochemical processes, such as microbial process (Holmberg and Ranta, 1982), bacterial growth systems (Dochain and Bastin, 1984), biomethanization process (Bastin et al., 1983), and fermentation processes (Boskovic, 1995). Research on parameter estimation for dynamic systems with nonlinear parametrization is a difficult and challenging area.

The parameter identification problem for microbial growth-process systems has been actively pursued in the literature (Holmberg and Ranta, 1982). Two basic methods have been developed to identify plant models described by Michaelis−Menten-type expressions. The first is to estimate the parameters of the aggregated model by manual curve fit-

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ting with the aid of sensitivity functions. The second is an optimization-based statistical estimation method. The main disadvantage of these techniques is that the estimated parameters are not necessarily unique, and therefore cannot be considered as biological characteristic of the processes (Holmberg and Ranta, 1982). In order to overcome the difficulty associated with the nonlinearly parametrized coefficients in Monod and Michaelis−Menten-type nonlinearities (Holmberg, 1983; Bastin et al., 1983; Dochain and Bastin, 1984), a different identification approach has been presented in (Dochain and Bastin, 1984) by considering the specific growth rate as a time-varying unknown parameter instead of an analytical function of the states. A least-squares algorithm has been developed to identify this time-varying growth rate. The advantage of the scheme is that the estimated parameters (namely the growth rate and a yield coefficient) have a clear physical meaning and can give useful real-time information about the plants. Although the parameter convergence cannot be proven theoretically, the simulation results show that the estimated parameter can follow the true time-varying growth rate effectively (Dochain and Bastin, 1984).

In this article, the adaptive parameter identification problem is considered for the nonlinearly parametrized chemical processes studied in the literature (Holmberg and Ranta, 1982; Dochain and Bastin, 1984; Boskovic, 1995, 1998). With the help of an integral-type Lyapunov function, an adaptive parameter estimation approach is constructed using Lyapunov's stability theory. A persistence of excitation condition, based on LaSalle's invariance theorem (LaSalle, 1968), is investigated for the proposed algorithm. Asymptotic convergence of the parameter estimation scheme is achieved when the system states satisfy a certain PE condition. The article is organized as follows. The following section describes the chemical plants and the identification problem under study. In the third section, an adaptive parameter estimation scheme is developed by using a parameter-dependent Lyapunov function. Persistency of excitation for the system signals is studied in the fourth section. Numerical simulations are included in the fifth section, followed by a brief conclusion in the sixth section.

System Description

We consider biochemical processes described by the dynamic model

$$\dot{x} = [\,\mu(x,s) - u\,]x \tag{1}$$

$$\dot{s} = -\frac{1}{Y}\mu(x,s)x + (s_0 - s)u \tag{2}$$

where states $x \in [0+\infty)$ and $s \in [0+\infty)$ denote biomass and substrate concentrations, respectively, u is the dilution rate, s_0 denotes the concentration of the substrate in the feed, and $Y > 0$ denotes the yield coefficient. The nonlinear function $\mu(x,s)$ denotes the growth rate of the process. The plants in Eqs. 1 and 2 can represent a large class of biochemical processes depending on the choice of the growth rate $\mu(x,s)$, such as microbial process, bacterial growth systems, bioreactor, distillation column, adsorption, and fermentation processes (Spriet, 1982; Holmberg and Ranta, 1982; Bastin et al., 1983; Boskovic, 1995).

There are many different models for $\mu(x,s)$ proposed in the literature (Spriet, 1982; Dochain and Bastin, 1984; Monod, 1942), for example

$$\mu(x,\, s) = \frac{\mu_m s}{K_s + s} \qquad \text{(Monod)} \tag{3}$$

$$\mu(x,\, s) = \frac{\mu_m s}{K_c x + s} \qquad \text{(Contois)} \tag{4}$$

$$\mu(x,s) = \frac{\mu_m K_0 s}{1 + K_1 s + K_2 s^2} \qquad \text{(Haldane)} \tag{5}$$

where $\mu_m > 0$ is the maximum value of the specific growth rate, and positive constants K_s, K_c, and K_0 to K_2 denote the coefficients for different growth-rate models. It can be seen that parameters K_s, K_c, K_1, and K_2 appear in their models nonlinearly, a major difficulty for parameter identification based on standard techniques (Holmberg and Ranta, 1982; Bastin et al., 1983).

In this article, we investigate the parameter estimation problem for the plants in Eqs. 1 and 2 with growth rate $\mu(x,s)$ represented by Monod's model (Eq. 3). Monod's model is one of the most commonly used models for growth kinetics. However, the scheme developed in this article is not limited to this model and can be easily extended to the plants with other growth-rate representations. Define the parameter vector $\theta = [\theta_\mu \;\; \theta_K \;\; \theta_Y]^T$, with

$$\theta_\mu = \mu_m, \qquad \theta_K = K_s, \qquad \theta_Y = \frac{\mu_m}{Y}$$

The plants in Eqs. 1−3 can be expressed as

$$\dot{x} = \frac{\theta_\mu s}{\theta_K + s}x - xu \tag{6}$$

$$\dot{s} = -\frac{\theta_Y s}{\theta_K + s}x + (s_0 - s)u \tag{7}$$

Parameter Estimation Method

Let $\hat{\theta}$ denote an estimate of the true parameter θ. Let $\hat{x}$ and $\hat{s}$ be estimates of x and s, respectively, by the dynamical system

$$\dot{\hat{x}} = f_x(w) - xu \tag{8}$$

$$\dot{\hat{s}} = f_s(w) + (s_0 - s)u \tag{9}$$

where $w = [x \; s \; \hat{x} \; \hat{s} \; \hat{\theta} \; u]^T$ and the nonlinear functions $f_s(w)$ and $f_s(w)$ are to be assigned later. Define $z_x = x - \hat{x}$ and $z_s = s - \hat{s}$. The error equations are

$$\dot{z}_x = \frac{\theta_\mu s}{\theta_K + s}x - f_x(w) \tag{10}$$

$$\dot{z}_s = -\frac{\theta_Y s}{\theta_K + s}x - f_s(w) \tag{11}$$

As mentioned in the literature (Holmberg and Ranta, 1982; Bastin et al., 1983), the major difficulty of this estimation problem is that the unknown parameter θ_K appears nonlinearly in Eqs. 10 and 11. To overcome this difficulty, the following integral-type Lyapunov function candidate, developed originally in Zhang et al. (2000), is used

$$V_z = \int_0^{z_x} (\theta_K + s)\sigma \, d\sigma + \int_0^{z_s} (\theta_K + \sigma + \hat{s})\sigma \, d\sigma \quad (12)$$

Since $s \in [0 + \infty)$, we know that $\theta_K + s \geq \theta_K > 0$. Hence, the parameter-dependent function, V_z, is positive definite and radially unbounded, and satisfies $V_z \to 0$ as $z_x, z_s \to 0$. The time derivative of V_z is

$$\dot{V}_z = z_x(\theta_K + s)\dot{z}_x + \int_0^{z_x} \dot{s}\sigma \, d\sigma + z_s(\theta_K + s)\dot{z}_s + \int_0^{z_s} \dot{\hat{s}}\sigma \, d\sigma$$

Substituting Eqs. 7, 9, 10, and 11 into the preceding equation, we have

$$\begin{aligned}\dot{V}_z &= z_x[\theta_\mu sx - (\theta_K + s)f_x(w)] - \frac{z_x^2}{2}\left[\frac{\theta_Y s}{\theta_K + s}x - (s_0 - s)u\right] \\ &\quad - z_s[\theta_Y sx + (\theta_K + s)f_s(w)] + \frac{1}{2}z_s^2[f_s(w) + (s_0 - s)u] \\ &= z_x\left[\theta_\mu sx + \frac{1}{2}(s_0 - s)uz_x - (\theta_K + s)f_x(w)\right] - \frac{\theta_Y sx}{2(\theta_K + s)}z_x^2 \\ &\quad + z_s\left[-\theta_Y sx + \frac{1}{2}(s_0 - s)uz_s - \left(\theta_K + \frac{s + \hat{s}}{2}\right)f_s(w)\right] \quad (13)\end{aligned}$$

Now choose

$$f_x(w) = \frac{1}{\hat{\theta}_K + s}\left\{\hat{\theta}_\mu sx + \left[\frac{1}{2}(s_0 - s)u + k_x\right]z_x\right\} \quad (14)$$

$$f_s(w) = \frac{2}{2\hat{\theta}_K + s + \hat{s}}\left\{-\hat{\theta}_Y sx + \left[\frac{1}{2}(s_0 - s)u + k_s\right]z_s\right\} \quad (15)$$

where constant gains $k_x, k_s > 0$, and $\hat{\theta}_K$, $\hat{\theta}_\mu$, and $\hat{\theta}_Y$ are the estimates of θ_K, θ_μ, and θ_Y, respectively. Define

$$\tilde{\theta}_K = \theta_K - \hat{\theta}_K, \qquad \tilde{\theta}_\mu = \theta_\mu - \hat{\theta}_\mu, \qquad \tilde{\theta}_Y = \theta_Y - \hat{\theta}_Y$$

Using Eqs. 14 and 15, we may reexpress Eq. 13 as

$$\begin{aligned}\dot{V}_z &= z_x\left[\theta_\mu sx + \frac{1}{2}(s_0 - s)uz_x - (\tilde{\theta}_K + \hat{\theta}_K + s)f_x(w)\right] \\ &\quad - \frac{\theta_Y sx}{2(\theta_K + s)}z_x^2 + z_s\left[-\theta_Y sx + \frac{1}{2}(s_0 - s)uz_s\right. \\ &\quad \left. - \left(\tilde{\theta}_K + \hat{\theta}_K + \frac{s + \hat{s}}{2}\right)f_s(w)\right] = z_x[\tilde{\theta}_\mu sx - \tilde{\theta}_K f_x(w)] \\ &\quad - \left[k_x + \frac{\theta_Y sx}{2(\theta_K + s)}\right]z_x^2 + z_s[-\tilde{\theta}_Y sx - \tilde{\theta}_K f_s(w)] - k_s z_s^2 \\ &= -\left[k_x + \frac{\theta_Y sx}{2(\theta_K + s)}\right]z_x^2 - k_s z_s^2 + \tilde{\theta}_\mu sxz_x - \tilde{\theta}_Y sxz_s \\ &\quad - \tilde{\theta}_K[f_x(w)z_x + f_s(w)z_s] \quad (16)\end{aligned}$$

The next task is to find adaptive estimation laws to update $\hat{\theta}_K$, $\hat{\theta}_\mu$, and $\hat{\theta}_Y$. We choose an augmented Lyapunov function candidate

$$V_s = V_z + \frac{1}{2}\left(\frac{\tilde{\theta}_K^2}{\gamma_1} + \frac{\tilde{\theta}_\mu^2}{\gamma_2} + \frac{\tilde{\theta}_Y^2}{\gamma_3}\right)$$

with constant gains $\gamma_1, \gamma_2, \gamma_3 > 0$. Its time derivative is

$$\begin{aligned}\dot{V}_s &= \dot{V}_z + \frac{1}{\gamma_1}\tilde{\theta}_\mu\dot{\tilde{\theta}}_\mu + \frac{1}{\gamma_2}\tilde{\theta}_K\dot{\tilde{\theta}}_K + \frac{1}{\gamma_3}\tilde{\theta}_Y\dot{\tilde{\theta}}_Y \\ &= -\left[k_x + \frac{\theta_Y sx}{\theta_K + s}\right]z_x^2 - k_s z_s^2 + \tilde{\theta}_\mu\left(sxz_x - \frac{\dot{\hat{\theta}}_\mu}{\gamma_1}\right) \\ &\quad - \tilde{\theta}_Y\left(sxz_s + \frac{\dot{\hat{\theta}}_Y}{\gamma_2}\right) - \tilde{\theta}_K\left[f_x(w)z_x + f_s(w)z_s + \frac{\dot{\hat{\theta}}_K}{\gamma_3}\right] \quad (17)\end{aligned}$$

Consequently, the following parameter updating laws are chosen

$$\dot{\hat{\theta}}_\mu = \gamma_1 sxz_x \quad (18)$$

$$\dot{\hat{\theta}}_Y = -\gamma_2 sxz_s \quad (19)$$

$$\dot{\hat{\theta}}_K = \begin{cases} -\gamma_3[f_x(w)z_x + f_s(w)z_s], & \text{if } \hat{\theta}_K > \epsilon \\ & \text{or } \hat{\theta}_K = \epsilon \text{ and } f_x(w)z_x \\ & + f_s(w)z_s \leq 0 \\ 0, & \text{otherwise,} \end{cases} \quad (20)$$

with initial value $\hat{\theta}_K(0) \geq \epsilon$ and a sufficiently small $\epsilon > 0$ satisfying $\theta_K > \epsilon$. The tuning law (Eq. 20) is a projection algorithm. It can be seen that for the case of $\hat{\theta}_K > \epsilon$ or $\hat{\theta}_K = \epsilon$ and $f_x(w)z_x + f_s(w)z_s \leq 0$, the projection algorithm (Eq. 20) suggests that $\dot{\hat{\theta}}_K = -\gamma_3[f_x(w)z_x + f_s(w)z_s]$, which leads to

$$\tilde{\theta}_K\left[f_x(w)z_x + f_s(w)z_s + \frac{\dot{\hat{\theta}}_K}{\gamma_3}\right] = 0 \quad (21)$$

For the case of $\hat{\theta}_K = \epsilon$ and $f_x(w)z_x + f_s(w)z_s > 0$, the learning algorithm (Eq. 20) leads to $\dot{\hat{\theta}}_K = 0$, which implies that

$$\begin{aligned}&\tilde{\theta}_K\left[f_x(w)z_x + f_s(w)z_s + \frac{\dot{\hat{\theta}}_K}{\gamma_3}\right] \\ &\quad = (\theta_K - \epsilon)[f_x(w)z_x + f_s(w)z_s] > 0 \quad (22)\end{aligned}$$

Combining Eqs. 21 and 22, we know that $\tilde{\theta}_K[f_x(w)z_x + f_s(w)z_s + \dot{\hat{\theta}}_K/\gamma_3] \geq 0$. It also can be seen from Eq. 20 that $\hat{\theta}_K(0) \geq \epsilon$ and $\dot{\hat{\theta}}_K \geq 0$ for $\hat{\theta}_K = \epsilon$. We conclude that $\hat{\theta}_K \geq \epsilon$ for all times. Substituting updating laws (Eqs. 18–20) into Eq. 17, we obtain

$$\dot{V}_s \leq -\left(k_x + \frac{\theta_Y sx}{\theta_K + s}\right)z_x^2 - k_s z_s^2$$

Noting that $\theta_Y, \theta_K > 0$ and $x, s \geq 0$, we have $\dot{V}_s \leq -k_x z_x^2 - k_s z_s^2$. By applying LaSalle−Yoshizawa's Theorem (Krstic et al., 1995), it is concluded that all signals in the estimation system (Eqs. 8, 9, and 18−20) are bounded. In addition, the state estimate errors z_x and z_s converge to zero as time goes to infinity. It should be noticed that $\lim_{t\to\infty} z_x(t)=0$ and $\lim_{t\to\infty} z_s(t)=0$ do not guarantee that the parameter estimation errors $\tilde{\theta}=[\tilde{\theta}_K\ \tilde{\theta}_\mu\ \tilde{\theta}_Y]^T$ go to zero as $t\to\infty$. In the following, we study the parameter convergence of the proposed adaptive identification algorithm.

PE Condition for Parameter Convergence

By LaSalle's Invariance Theorem (Krstic et al., 1995), the error vector $(z_x, z_s, \tilde{\theta})$ converges to the largest invariant set, M, of the error system (Eqs. 10, 11, and 18−20) contained in the set

$$E=\left\{\left(z_x, z_s, \tilde{\theta}\right)\in R^5 | z_x=0,\ z_s=0\right\}$$

The purpose of the following is to study the invariant set, M, to obtain the conditions under which parameter convergence can be achieved. Since z_x and z_s converge to zero, we know that

$$\int_0^\infty \dot{z}_x\, dt = z_x(\infty)-z_x(0) = -z_x(0)$$

$$\int_0^\infty \dot{z}_s\, dt = z_s(\infty)-z_s(0) = -z_s(0)$$

This implies that $\dot{z}_x$ and $\dot{z}_s$ are integrable. It follows from the error equations (Eqs. 10 and 11) that $\ddot{z}_x$ and $\ddot{z}_s$ are functions of x, s, $\hat{x}$, $\hat{s}$, $\hat{\theta}$, u, and $\dot{u}$. Since $\hat{\theta}$, z_x and z_s are bounded, then it follows that if x, s, u, and $\dot{u}$ are bounded, $\ddot{z}_x$ and $\ddot{z}_s$ are bounded. This implies the uniform continuity of $\dot{z}_x$ and $\dot{z}_s$. By Barbalat's lemma (Ioannou and Sun, 1996), we conclude that $\dot{z}_x, \dot{z}_s \to 0$ as $t\to\infty$.

On the invariant set M, we have $z_x = z_s \equiv 0$ and $\dot{z}_x=\dot{z}_s\equiv 0$. Setting $z_x=z_s=\dot{z}_x=\dot{z}_s=0$, Eqs. 10 and 11 lead to

$$\frac{\theta_\mu s}{\theta_K+s}x = \lim_{\hat{x}\to x} f_x(w) = \frac{\hat{\theta}_\mu s}{\hat{\theta}_K+s}x, \qquad \forall\left(z_x, \tilde{\theta}\right)\in M \quad (23)$$

$$\frac{\theta_Y s}{\theta_K+s}x = \lim_{\hat{s}\to s} f_s(w) = \frac{\hat{\theta}_Y s}{\hat{\theta}_K+s}x, \qquad \forall\left(z_s, \tilde{\theta}\right)\in M \quad (24)$$

Hence

$$\frac{\theta_\mu sx}{\theta_K+s} = \frac{\hat{\theta}_\mu sx}{\hat{\theta}_K+s} = \frac{\left(\theta_\mu-\tilde{\theta}_\mu\right)sx}{\theta_K-\tilde{\theta}_K+s}, \qquad \forall\left(z_x, \tilde{\theta}\right)\in M \quad (25)$$

$$\frac{\theta_Y sx}{\theta_K+s} = \frac{\hat{\theta}_Y sx}{\hat{\theta}_K+s} = \frac{\left(\theta_Y-\hat{\theta}_Y\right)sx}{\theta_K-\tilde{\theta}_K+s}, \qquad \forall\left(z_s,\tilde{\theta}\right)\in M \quad (26)$$

Through some manipulations, we have

$$\left[\tilde{\theta}_\mu\ \tilde{\theta}_K\right]\begin{bmatrix}\theta_K+s\\ -\theta_\mu\end{bmatrix}sx=0, \qquad \left[\tilde{\theta}_\mu\ \tilde{\theta}_Y\right]\begin{bmatrix}\theta_K+s\\ -\theta_Y\end{bmatrix}sx=0,$$

$$\forall\left(z_x, z_s, \tilde{\theta}\right)\in M \quad (27)$$

Therefore, the largest invariant set, M, in E is

$$M=\left\{\left(z_x, z_s, \tilde{\theta}\right)\in R^5 | z_x=z_s=0,\ \left[\tilde{\theta}_\mu\ \tilde{\theta}_K\right]\begin{bmatrix}\theta_K+s\\ -\theta_\mu\end{bmatrix}sx=0,\right.$$

$$\left.\left[\tilde{\theta}_\mu\ \tilde{\theta}_Y\right]\begin{bmatrix}\theta_K+s\\ -\theta_Y\end{bmatrix}sx=0\right\}$$

It follows from Eq. 27 that

$$\left[\tilde{\theta}_\mu\ \tilde{\theta}_K\right]\Phi_\mu(x,s)\begin{bmatrix}\tilde{\theta}_\mu\\ \tilde{\theta}_K\end{bmatrix}=0, \qquad \left[\tilde{\theta}_\mu\ \tilde{\theta}_Y\right]\Phi_Y(x,s)\begin{bmatrix}\tilde{\theta}_\mu\\ \tilde{\theta}_Y\end{bmatrix}=0,$$

$$\forall\left(z_x, z_s, \tilde{\theta}\right)\in M \quad (28)$$

where

$$\Phi_\mu(x,s)=s^2x^2\begin{bmatrix}(\theta_K+s)^2 & -\theta_\mu(\theta_K+s)\\ -\theta_\mu(\theta_K+s) & \theta_\mu^2\end{bmatrix} \quad (29)$$

$$\Phi_Y(x,s)=s^2x^2\begin{bmatrix}(\theta_K+s)^2 & -\theta_Y(\theta_K+s)\\ -\theta_Y(\theta_K+s) & \theta_Y^2\end{bmatrix} \quad (30)$$

If $\Phi_\mu(x, s)$ and $\Phi_Y(x, s)$ are positive definite, then we can conclude that $\tilde{\theta}=0$. However, it is impossible to satisfy this condition because the matrices $\Phi_\mu(x, s)$ and $\Phi_Y(x, s)$ are singular at any given time. We consider the integrals of $\Phi_\mu(x, s)$ and $\Phi_Y(x, s)$ for $t\to\infty$. It follows from Eq. 27 that

$$\lim_{t\to\infty}\frac{1}{T_0}\int_t^{t+T_0}\left[\tilde{\theta}_\mu\ \tilde{\theta}_K\right]\Phi_\mu(x,s)\begin{bmatrix}\tilde{\theta}_\mu\\ \tilde{\theta}_K\end{bmatrix}d\tau=0,$$

$$\forall\left(z_x, \tilde{\theta}\right)\in M \quad (31)$$

with positive constant T_0. By Eqs. 18−20 and $\lim_{t\to\infty} z_x, z_s = 0$, we know that $\lim_{t\to\infty}\dot{\hat{\theta}}=0$, which implies that $\tilde{\theta}_\mu$ $\tilde{\theta}_K$, and $\tilde{\theta}_Y$ are constants when $t\to\infty$. Therefore

$$\left[\tilde{\theta}_\mu\ \tilde{\theta}_K\right]\left\{\lim_{t\to\infty}\frac{1}{T_0}\int_t^{t+T_0}\Phi_\mu(x,s)\,d\tau\right\}\begin{bmatrix}\tilde{\theta}_\mu\\ \tilde{\theta}_K\end{bmatrix}=0,$$

$$\forall\left(z_x, \tilde{\theta}\right)\in M. \quad (32)$$

We now are ready to present the persistence of excitation condition for parameter convergence. If states $x(t)$ and $s(t)$ satisfy the following inequality

$$\lim_{t\to\infty}\frac{1}{T_0}\int_t^{t+T_0}\Phi_\mu(x,s)\,d\tau \geq c_0 I \quad (33)$$

with some $c_0>0$, then the estimated parameter error $\tilde{\theta}_\mu$ and $\tilde{\theta}_K$ converge to zero asymptotically. Similarly, if there exists a positive constant c_1 such that

$$\lim_{t\to\infty}\frac{1}{T_0}\int_t^{t+T_0}\Phi_Y(x,s)\,d\tau \geq c_1 I \quad (34)$$

then, the estimated parameters $\hat{\theta}_\mu$ and $\hat{\theta}_Y$ are guaranteed to converge to their true values.

Theorem 4.1. Consider the parameter estimation system consisting of the plants (given in Eqs. 1 and 2, estimator Eqs. 8 and 9), and updating laws Eqs. 18–20. If system variables x and s, and input signal u are bounded, and the system states satisfy PE conditions given in Eqs. 33 and 34, then $\lim_{t \to \infty} \hat{\theta}(t) = \theta$.

Remark 4.1. In order to achieve the parameter convergence, the PE conditions (given in Eqs. 33 and 34) indicate that the variations of system states $x(t)$ and $s(t)$ should make the integrals of matrices $\Phi_\mu(x, s)$ and $\Phi_Y(x, s)$ positive definite. Let us check the PE condition for a special case, $x = c_x$ and $s = c_s$ with constants c_x, $c_s > 0$. In this case, we have

$$\lim_{t \to \infty} \frac{1}{T_0} \int_t^{t+T_0} \Phi_\mu(x, s)\, d\tau = c_x^2 c_s^2 \begin{bmatrix} (\theta_K + c_s)^2 & -\theta_\mu(\theta_K + c_s) \\ -\theta_\mu(\theta_K + c_s) & \theta_\mu^2 \end{bmatrix}$$

Clearly, the preceding matrix is not positive definite, since its determinant is zero. Hence, the estimated parameters cannot be guaranteed to converge to their true values for the regulation case. A practical way to resolve the problem is to add an excitation signal to the control input such that system states contain sufficient excitation for the parameter convergence.

Remark 4.2. Compared with the standard PE condition in the literature (Ioannou and Sun, 1996), which requires

$$\frac{1}{T_0} \int_t^{t+T_0} \Phi(\tau) \Phi^T(\tau)\, d\tau \geq c_0 I, \qquad \forall t \geq 0$$

for the system regressor matrix $\Phi(\tau)$, the proposed PE conditions (Eqs. 33 and 34) are weakened, since Eqs. 33 and 34 need only hold for $t \to \infty$. Accordingly, only an asymptotic parameter convergence is guaranteed by Theorem 4.1, instead of the exponential convergence obtained in the references (Narendra and Annaswamy, 1989; Ioannou and Sun, 1996).

Remark 4.3. It should be noticed that Monod's growth model used in this work is only one of many possible approximation models for an actual process under certain conditions. Its parameters may not be intrinsic or even unique. In some applications, an actual data set may fit Monod's model very well under different parameter sets. Hence, in the development of parameter identification algorithms, additional precautions should be taken to ensure the uniqueness of the model parameters and the sensitivity of the model predictions to its parameters. Although this article considers the chemical processes represented by Monod's growth model only, the proposed design scheme can be easily extended to other more dynamically rich models. For instance, if the growth rate $\mu(x, s)$ is expressed by Contois' model given in Eq. 4, we can choose the following Lyapunov function

$$V_z = \int_0^{z_x} [K_c(\sigma + \hat{x}) + s] \sigma\, d\sigma + \int_0^{z_s} (K_c x + \sigma + \hat{s}) \sigma\, d\sigma \qquad (35)$$

If it is expressed by the Haldane's model shown in Eq. 5, the following Lyapunov function can be utilized

$$V_z = \int_0^{z_x} \left[1 + K_1 s + K_2 s^2\right] \sigma\, d\sigma + \int_0^{z_s} \left[1 + K_1(\sigma + \hat{s}) + K_2(\sigma + \hat{s})^2\right] \sigma\, d\sigma \qquad (36)$$

By using a similar design procedure proposed in this article, parameter identification problems of the plants with Contois' model and Haldane's model are solvable.

Simulation Results

To show the effectiveness of the proposed design, simulation studies are performed using the experimental conditions provided in the work (Dochain and Bastin, 1984) for an anaerobic digestion process. As discussed in the reference (Dochain and Bastin, 1984), the model in Eqs. 1 and 2 is suitable for the description of the methanization stage in an anaerobic digestion process in which the input u is the influent acetic acid concentration (that is, the input pollution level), and the state s denotes the output pollution level. Further details on the anaerobic digestion process can be found in the literature (Bastin et al., 1983; Antunes and Installe, 1981). The following parameters and initial states are used in the simulation experiments

$$K_s = 0.4, \qquad \mu = 0.4, \qquad Y = 1/0.3636$$
$$s_0 = 2.0, \qquad x(0) = 0.069, \qquad s(0) = 0.13.$$

The state estimates $\hat{x}$ and $\hat{s}$ are generated by

$$\dot{\hat{x}} = \frac{1}{\hat{\theta}_K + s} \left\{ \hat{\theta}_\mu s x + \left[\frac{1}{2}(s_0 - s) u + k_x \right] z_x \right\} - x u \qquad (37)$$

$$\dot{\hat{s}} = \frac{2}{2\hat{\theta}_K + s + \hat{s}} \left\{ -\hat{\theta}_Y s x + \left[\frac{1}{2}(s_0 - s) u + k_s \right] z_s \right\} + (s_0 - s) u \qquad (38)$$

with $\hat{x}(0) = \hat{s}(0) = 0.5$. The input signal $u(t)$ (see Figure 1a) is chosen as a multilevel sequence filtered by a low-pass filter $1/(0.2s + 1)$ (which smoothes the input signal for an easy application).

Case 1: slow adaptive learning rates

The estimated parameters $\hat{\theta}_K$, $\hat{\theta}_\mu$, and $\hat{\theta}_Y$ are updated by Eqs. 18–20, with $\epsilon = 0.01$, $\hat{\theta}_K(0) = 0.8$, $\hat{\theta}_\mu(0) = \hat{\theta}_Y(0) = 0$, and

$$\gamma_1 = 0.01, \qquad \gamma_2 = 0.01, \qquad \gamma_3 = 0.04$$

The gains in Eqs. 37 and 38 are $k_x = 0.05$ and $k_s = 0.05$. The system states and their estimates are plotted in Figure 1b and 1c, and the estimates $\hat{\theta}_K$, $\hat{\theta}_\mu$, and $\hat{\theta}_Y$ are shown in Figure 2a–2c, respectively. It can be seen that although the estimated parameters are close to their true values at $t = 200$, they have not converged. Parameter convergence is only ob-

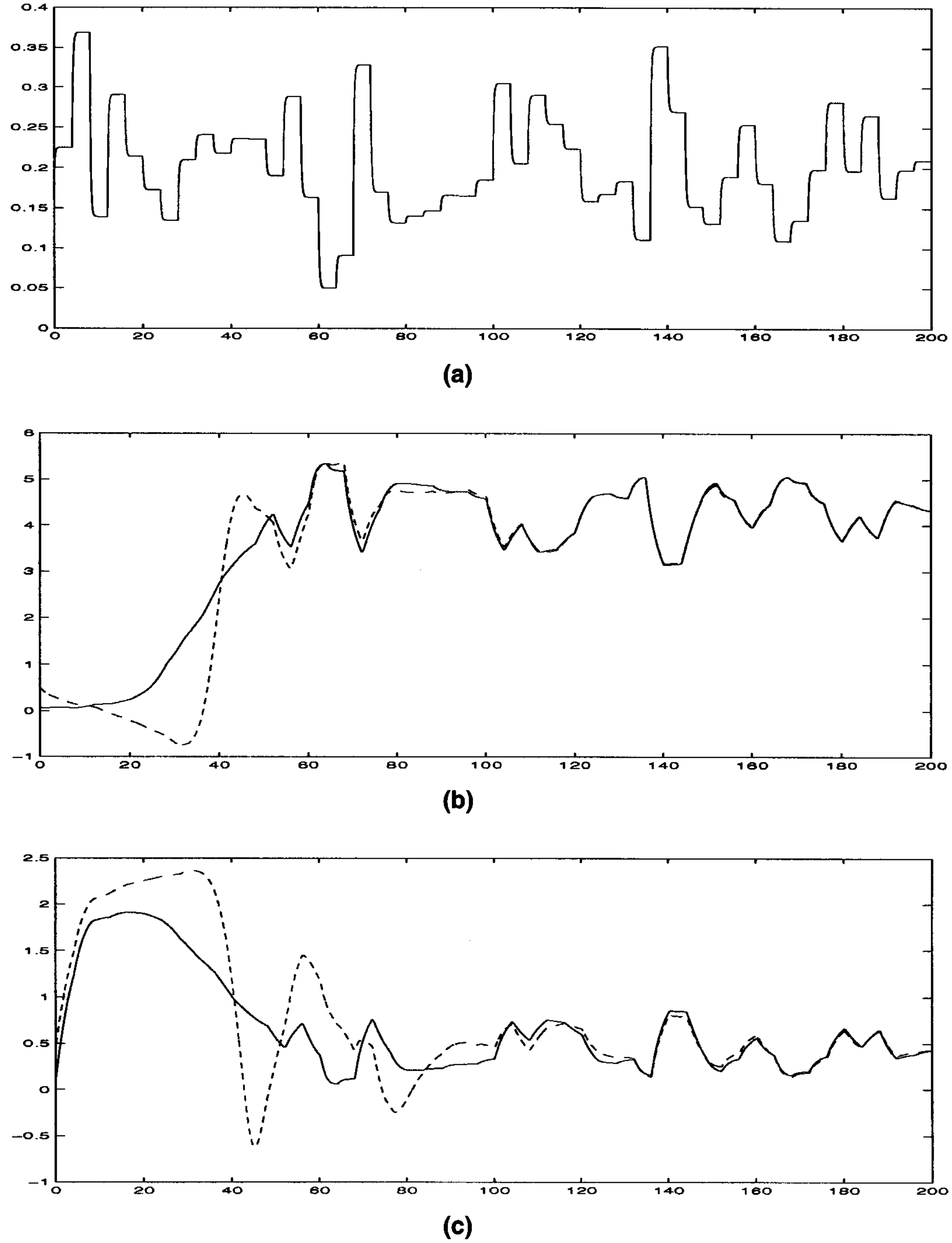

Figure 1. Input signal and system states for Case 1.

(a) Input signal $u(t)$; (b) states x (—) and $\hat{x}$ (−−); (c) states s (—) and $\hat{s}$ (−−).

served for simulations performed over much larger time intervals ($t = 2000$). To investigate this phenomenon, we plot Monod growth rate $\mu(x,\ s) = \theta_\mu s/(\theta_K + s)$ and its estimate $\hat{\mu}(x,\ s) = \hat{\theta}_\mu s/(\hat{\theta}_K + s)$ in Figure 2d. It can be seen that after $t = 100$, the error between $\mu(x,\ s)$ and its estimate $\hat{\mu}(x,\ s)$ is very small even if parameter estimation errors still exist. This implies that the Monod growth model $\mu(x,\ s)$ is not sensitive to parameter variations when the parameters and the states are within certain ranges. One way to get a more accurate estimation is to increase the level of excitation of the input signal. Another way is to find better operating points where $\mu(x,\ s)$ is sensitive to variations of model parameter values.

Case 2: fast adaptive learning rates

Next, we investigate the sensitivity of the estimation scheme to the design parameters by increasing the learning rates to

$$\gamma_1 = 0.1, \qquad \gamma_2 = 0.1, \qquad \gamma_3 = 0.4$$

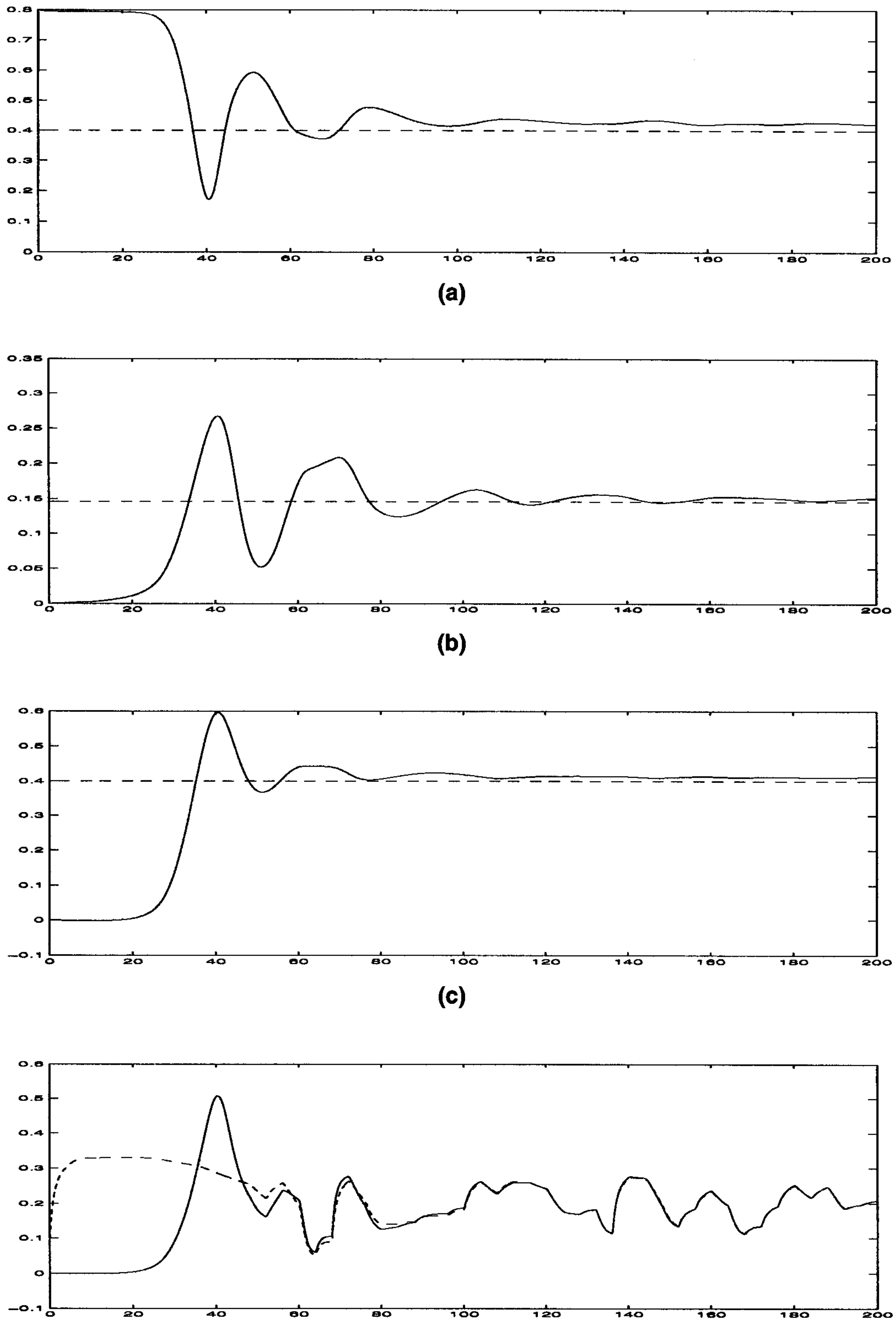

Figure 2. Parameter convergence of Case 1 (slow adaptation rates).

(a) Parameter estimate $\hat{\theta}_K$ (—) and its true value θ_K (--); (b) parameter estimate $\hat{\theta}_Y$ (—) and its true value θ_Y (--); (c) parameter estimate $\hat{\theta}_\mu$ (—) and its true value θ_μ (--); (d) growth rate $\mu(x, s)$ (--) and its estimate $\hat{\mu}(x, s)$ (—).

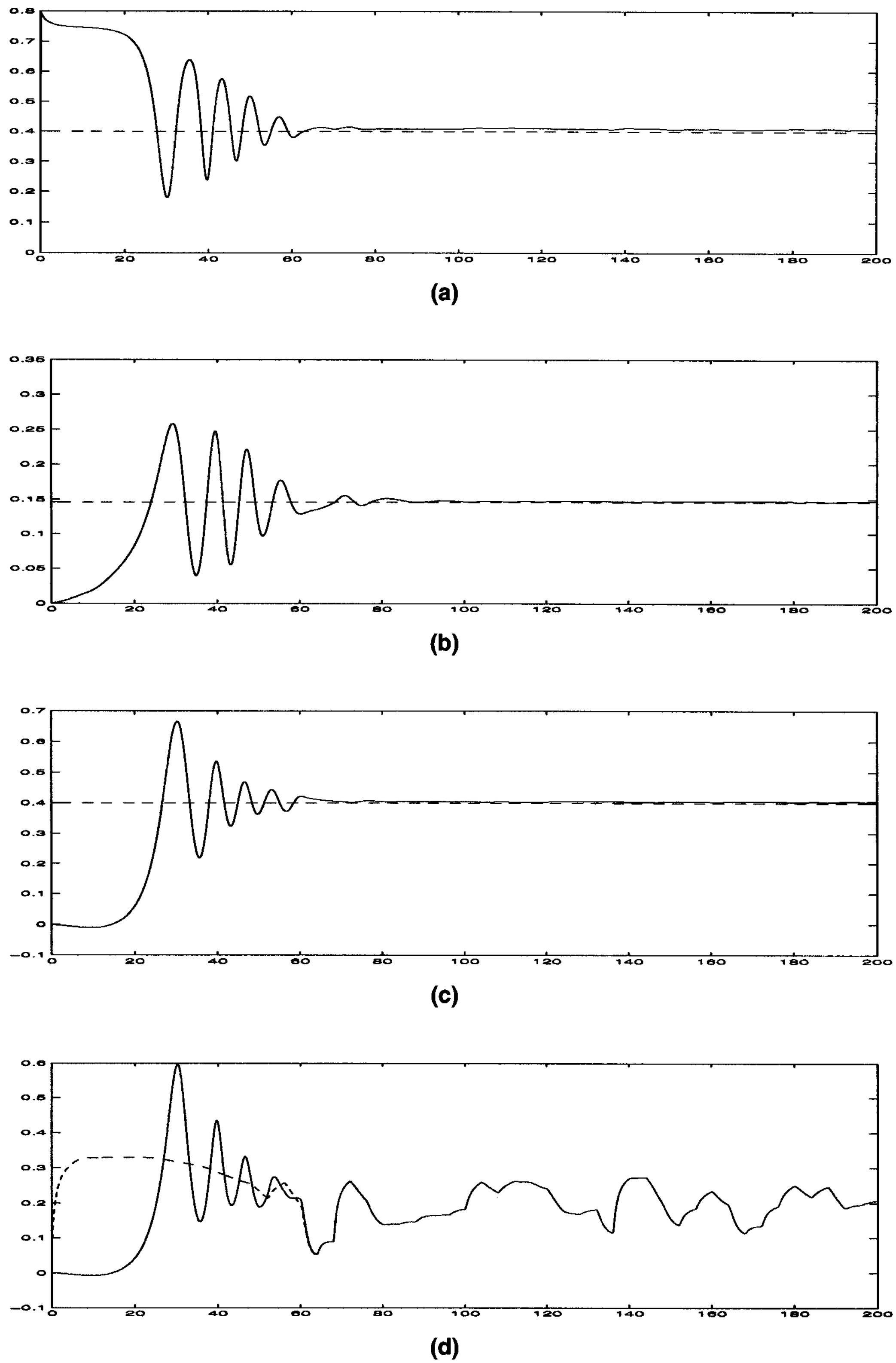

Figure 3. Parameter convergence of Case 2 (fast adaptation rates).

(a) Parameter estimate $\hat{\theta}_K$ (—) and its true value θ_K (--); (b) parameter estimates θ_Y (—) and its true value θ_Y (--); (c) parameter estimate $\hat{\theta}_\mu$ (—) and its true value θ_μ (--); (d) growth rate $\mu(x, s)$ (--) and its estimate $\hat{\mu}(x, s)$ (—).

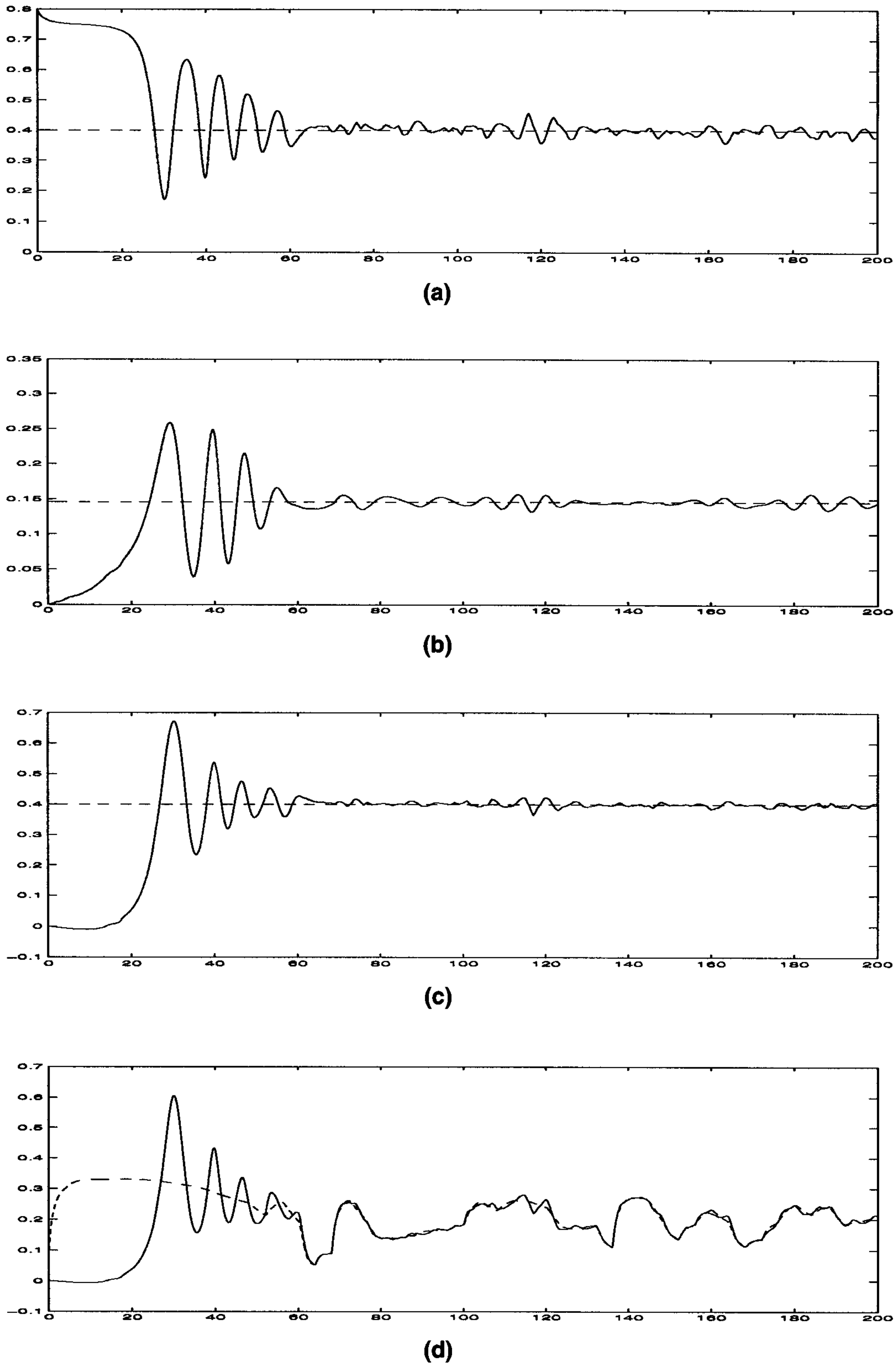

Figure 4. Parameter identification of Case 3 (existence of measurement noises).

(a) Parameter estimate $\hat{\theta}_K$ (—) and its true value θ_K (——); (b) parameter estimate $\hat{\theta}_Y$ (—) and its true value θ_Y (——); (c) parameter estimate $\hat{\theta}_\mu$ (—) and its true value θ_μ (--); (d) growth rate $\mu(x, s)$ (--) and its estimate $\hat{\mu}(x, s)$ (—).

The gains in Eqs. 37 and 38 are set to $k_x = 0.1$ and $k_s = 0.1$. Figures 3a–3d show the simulation results for the fast adaptation case. Comparing with the plots in Figures 2a–2d, we see that faster parameter convergence is obtained as adaptive learning rates increase. However, larger oscillations are observed during $20 < t < 60$ due to the high adaptation gains.

Case 3: noisy measurements

In practical applications, only noisy measurements of system states are available. To investigate the effects of the noises, we repeat the simulation given earlier by adding 5% random noises to the state measurements. Figures 4a–4d show the identification results. It is shown that the estimated parameters approach the neighborhood of their true values. The oscillations observed during $100 < t < 200$ indicate that the algorithms with high adaptation gains are sensitive to measurement noises. Decreasing parameter learning rates may help to reduce the sensitivity of the algorithms and alleviate the oscillations of the estimated parameters.

Comparing with the experiment result of Dochain and Basin (1984), we see that a faster parameter estimation convergence is achieved using the proposed identification scheme. Furthermore, the proposed algorithms guarantee the convergence of the model parameters. Note that both biomass and substrate measurements are assumed to be available in the proposed approach. This may partially account for the difference in performance.

Conclusion

An adaptive parameter identification algorithm has been studied for a class of nonlinearly parametrized plants, which covers a large class of biochemical processes. Lyapunov's design technique has been used to construct the estimation algorithm and develop the persistence of the excitation condition. A new feature of the proposed design is the use of a novel Lyapunov function containing the unknown parameter, which is instrumental in alleviating the difficulty associated with the identification of Monod's model coefficients. Simulation studies show that the proposed approach can efficiently identify the yield coefficient, the maximum value of the growth rate, and Monod's coefficient for the bacterial growth systems.

Acknowledgments

This work was supported by the National Sciences and Engineering Research Council of Canada and the Canadian Foundation for Innovation. The authors wish to thank the anonymous reviewers for their comments and suggestions for improving the paper.

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Manuscript received Feb. 21, 2001, and revision received Aug. 8, 2001.